A Review of Donald G. Saari's "Are Individual Rights Possible?"
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"Are Individual Rights Possible?" by Donald G. Saari discusses Amartya Sen's analysis of the problems with decision making based on individual choices, or preferences, in society. Sen's theory, which offers insight into a very interesting projection of mathematics onto the social sciences, incorporates the ideas discussed in Math 152 such as transitivity and mapping with restricted domains. This interesting application begs the question of whether or not the absolute freedom of choice has a place in society. While the likely conclusion is that it does, Sen's surprising, if not unsettling argument suggests that it actually makes group decision-making impossible. As Saari explains, while Sen's theory, so directly related to our daily lives, has applications in fields ranging from philosophy to mathematics, experts in any of the fields have yet to respond with a viable means of escaping the trap that Sen describes.

Sen's theorem summarizing this assertion is as follows:
Theorem (Sen). *With $n \geq 3$ alternatives and at least two voters, no procedure can satisfy (U), (P), and (ML) and avoid cyclic outcomes.*

In order to understand Sen's theorem, it is important to discuss and fully understand the assumptions he makes. The assumptions, proposed to exist in a society which offers n>3 choices, are:

(U) ***Unrestricted domain***. *Each individual can rank the alternatives in any desired transitive manner.*
This assumption relates the transitivity to preferences, such that if an individual prefers a to b and b to c, then a is preferred to c. Voters who upload such preferences are said to be "rational," while those who don't are referred to as "primitive" for the purposes of this paper.

(P) ***Pareto***. *If every individual prefers a>b, then society prefers a>b.*
This assumption is very straightforward; if everyone shares the same preference, then that preference is clearly representative of society as a whole. (The notation "a>b" means that choice a is preferred to choice b.)

(ML) ***Minimal Liberalism***. *There are at least two individuals who are decisive over different pairs of alternatives; the decisive voter's personal ranking for the assigned pair determines society's ranking of the pair.*
This third assumption encompasses the notion of "individual liberty," suggesting that each individual retains autonomy over his or her choices. Only you, for example, can decide what *you* want to eat for dinner tonight, and only Kate can decide what *she* wants to eat for dinner.

While these assumptions appear natural and straightforward, a closer examination of them, as proposed by Sen's theorem, suggests that the combination of those three conditions make decision-making impossible in society.

This paper discusses Sen's proof through the use of examples illustrating the results of pairwise ranked choices. The heart of the problem lies in the creation of cycles that results from the inclusion of all of these assumptions. A cycle occurs in such a case, for example, when A>B, B>C, but C>A. Note, this relationship is *not* transitive; in the transitive case, given the first two rankings, one would expect A>C.

A case constructed by Sen, depicted below, illustrates a situation in which our three initial assumptions generate a cycle. In the following table, "Voter 1" is decisive over {a, b}, meaning that his ranking represents society's ranking for a versus b, and "Voter 2" is decisive over {b,c}.

TABLE 1: Sen's first example

Voter	Choice $\{a, b\}$	$\{b, c\}$	$\{a, c\}$
1	$a \succ b$	—	$c \succ a$
2	—	$b \succ c$	$c \succ a$
Others	—	—	$c \succ a$

In this example, we see that Voter 1 prefers a to b, and voter 2 prefers b to c, while all three voters prefer c to a. Let us now examine the omitted entries, which were not included in the example because they are insignificant, given each voter's decisive domain. Since voter 1 prefers a>b and c>a, transitivity would suggest that his choice {b,c} would be c>b. By similar logic, then we can say that Voter 2 prefers b>a.

At this point, we can use our established conditions to determine society's overall preferences. For {a, b}, we know that a>b (since Voter 1 is decisive for a>b), and for {b,c}, we have b>c. Since all voters prefer c>a for {a,c}, that preference holds for society given the Pareto condition. We are left, then, with a cycle; transitivity would suggest that if a>b and b>c, then a>c, but rather, society ranks c above a.

To see this in application, let's consider a real-world example. Suppose a husband and wife are buying a car for their family, and are choosing among three models, a Lexus, an Mercedes, and a Rolls-Royce. The husbands preferences are: Rolls-Royce > Lexus > Mercedes, while the wife's preferences are: Mercedes > Rolls-Royce > Lexus. Given the decisiveness established in the chart, then, the family as a whole would prefer the Lexus to the Mercedes, the Mercedes to the Rolls-Royce, but the Rolls-Royce to the Lexus, thus creating a cycle. Though one might wonder if the overlap of choice "b" in each decisive voter's domain ({a, **b**} and {**b**, c} for Voters 1 and 2, respectively), Sen goes on to construct a similar situation, introducing a choice d, in which there is no overlap between the decisive voters' decisions.

This paper goes on to discuss possible (yet not optimal) resolutions to the problem of cycles. Sen proposes restricting the domain required by ML such that the mapping of that domain, a function that expresses society's ranking of each choice, is non-cyclical, or defined. To determine allowable domains, Sen suggests creating lists of all of the possible rankings choices each voter could make, ignoring the requirement of transitivity, and then examining various ML voting procedures with respect to the domain of each possible ranking for each voter. The result is that the only procedures that can resolve Sen's conflict are those which allow for a domain inclusive of non-transitive voter preferences. That is to say that allowable procedures are those which are acceptable in "primitive" societies. This is exclusive of commonly used procedures today, such as the plurality vote, in which each voter votes for his top-ranked candidate, and a Borda count, in which case each rank is given a point-value, with higher ranks being worth more points, and then the scores are summed for each choice. The flaw in each of these procedures, making them susceptible to Sen's problem, is that they each assume transitivity in voter preferences. Indeed, one could restore the property of transitivity and voter rationality, but in order to avoid cyclic outcomes, the preferences would have to be contrived, therefore eliminating the voter's freedom of choice.

Reading this article at a time when campaigns for the Undergraduate Council presidency are in full swing prompted me to explore the voting mechanism used for these elections. Interestingly, while voters rank their choices in order of preference, the votes are not counted by a Borda count, as described above, but rather by the "single transferable vote" procedure. In this case, a voter's vote counts first towards his top choice. Candidates who receive beyond a certain threshold of votes move on to the next round; in these rounds, if a voter's top candidate did not make it through, his vote is transferred to the highest ranked candidate on his list who has made it through to that round. This mechanism is designed so that each voter's vote does not go to waste. The UC uses this largely because UC general elections involve candidates running for multiple seats; representatives of the UC Elections Commission were unsure why it is used over the plurality vote for Presidential elections, which offer only one seat. Though this system, which is also used in the city of Cambridge, Massachusetts, appears to be a very fair way of counting votes with rank-ordered preferences, the nature of the voting suggests that each individual must have transitively ranked choices. By the conclusions drawn in this article, then, this system, too, is susceptible to the entrapping creation of cycles.

This article, overall, presents a compelling theory which alerts the reader to the theoretical possibility of society arriving at a deadlock in terms of decision-making, escapable only by rejecting the very same principles of democracy on which our government is founded.